First determination of the quark mixing matrix element V_{tb} independent of assumptions of unitarity

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We present a new method for the determination of the Cabibbo-Kobayashi-Maskawa quark mixing matrix element $|V_{tb}|$ from electroweak loop corrections, in particular those affecting the process $Z \to b\bar{b}$. From a combined analysis of results from the LEP, SLC, Tevatron, and neutrino scattering experiments we determine $|V_{tb}| = 0.77^{+0.18}_{-0.24}$. This is the first determination of $|V_{tb}|$ which is independent of unitarity assumptions.

12.15.Ff, 12.15.Lk, 13.38.Dg, 14.65.Ha

I. INTRODUCTION

The Cabibbo-Kobayashi-Maskawa (CKM) matrix [1] describes the relationship between weak and mass eigenstates of quarks, assuming that there are three generations. By convention, up-type quarks are unmixed such that all the mixing is expressible in terms of the 3×3 unitary matrix V

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \tag{1}$$

where unprimed states denote mass eigenstates and primed ones denote weak eigenstates. The unitarity of the CKM matrix is an assumption which must be subjected to experimental verification by independent measurements of the elements. Non-unitarity of the matrix would be a clear signature for physics beyond the standard model, such as a fourth generation or non-universality of the quark couplings. Of particular interest in terms of possible new physics is the element V_{tb} which describes the coupling of the two heaviest quarks.

It is often assumed that $|V_{tb}| \approx 1$ although this element has never been determined without assumptions of unitarity. Assuming that there are only three generations and unitarity of the CKM matrix yields: $0.9989 < |V_{tb}| < 0.9993$ at the 90% confidence level [2]. Relaxing the assumption of three generations but maintaining that of unitarity yields $0 < |V_{tb}| < 0.9993$ at the 90% confidence level [2], while relaxing also that of unitarity leaves $|V_{tb}|$ unbounded.

In this paper we describe a new method for the determination of $|V_{tb}|$ from electroweak loop corrections, in particular to the process $Z \to b\bar{b}$. From a combined analysis of data from the LEP, SLC, Tevatron, and neutrino scattering experiments, we determine for the first time the value of $|V_{tb}|$, without any assumptions about the unitarity of the CKM matrix. The implications for other quantities, such as the the top mass, Higgs mass, and the strong coupling constant, are discussed.

II. EFFECTS OF V_{tb} ON ELECTROWEAK RADIATIVE CORRECTIONS

The precision of the electroweak data from the LEP and SLC experiments is sufficient to be sensitive to weak loop diagrams involving the top quark. The top quark appears in Z vacuum polarisation loops, thereby affecting all the Z partial widths, and in the GIM-suppressed vertex diagrams shown at the one-loop level in Fig. 1 which affect the Z partial width, $\Gamma_{b\bar{b}}$, for the process $Z \to b\bar{b}$. From fits to the Z electroweak parameters, the top quark mass has been determined to be $m_t = 158^{+14}_{-11}$ GeV [3], in agreement with the direct measurement from the Tevatron of $m_t = 175.6 \pm 5.5$ GeV [4].

Hitherto the theoretical treatments of weak loop corrections to Z decay processes have assumed that $|V_{tb}| = 1$; we relax this assumption. Following the treatment of Barbieri, Beccaria, Ciafaloni, Curci, and Viceré (BBCCV) [5], $\Gamma_{b\bar{b}}$ may be written as:

$$\Gamma_{b\bar{b}} = \frac{G_\mu m_Z^3}{8\pi\sqrt{2}} \rho R_{\mathrm{QED}} R_{\mathrm{QCD}} \sqrt{1 - \frac{4m_b^2}{m_Z^2}} \times \left[\left(g_{bV}^2 + g_{bA}^2\right)\left(1 + 2\frac{m_b^2}{m_Z^2}\right) - 6 g_{bA}^2 \frac{m_b^2}{m_Z^2} \right], \tag{2}$$

where G_μ is the Fermi constant, m_Z and m_b are the masses of the Z and the b-quark respectively, and ρ includes the effects of radiative corrections to the Z propagator. R_{QED} and R_{QCD}, which are approximately unity, describe the QED and QCD vertex corrections [6,7]. The couplings, g_{bV} and g_{bA} incorporate vertex corrections described by the parameter, τ, as follows:

$$g_{bA} = 1 + \tau \tag{3}$$

$$g_{bV} = 1 - \frac{4}{3}s^2 + \tau \tag{4}$$

$$\text{where}: \quad s^2 = \frac{1}{2}\left(1 - \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_\mu m_Z^2 \rho}}\right). \tag{5}$$

We have taken the results of the BBCCV calculations up to two-loops and included the effects of V_{tb} at the one-loop level, such that ρ is unchanged while τ is modified by a multiplicative factor of $|V_{tb}|^2$. In the limit $m_t \gg m_H$, τ is given by:

$$\tau = -2|V_{tb}|^2 x \left[1 + \frac{x}{3}\left(27 - \pi^2\right)\right], \tag{6}$$

where $x = G_\mu m_t^2/8\pi^2\sqrt{2}$. In the limit $m_t \ll m_H$, τ is given by:

$$\begin{aligned}\tau &= -2|V_{tb}|^2 x \\ &\times \left\{1 + \frac{x}{144}\left[\; 311 + 24\pi^2 + 282\log r + 90\log^2 r \right.\right. \\ &-4r\left(40 + 6\pi^2 + 15\log r + 18\log^2 r\right) \\ &+\frac{3r^2}{100}\left(24209 - 6000\pi^2 - 45420\log r\right. \\ &\left.\left.\left. -18000\log^2 r\right)\right]\right\}, \end{aligned} \tag{7}$$

where $r = m_t^2/m_H^2$. In the intermediate region ($m_t \approx m_H$) τ is described by a polynomial parametrisation of the full BBCCV calculation, as a function of m_t/m_H, multiplied by a factor of $|V_{tb}|^2$. At the two-loop level some diagrams are of the order $|V_{tb}|^4$. Our treatment of these to order $|V_{tb}|^2$ is justified by their relatively small contribution and the current sensitivity of the experimental data, as will be seen below.

III. DETERMINATION OF $|\mathbf{V_{tb}}|$ FROM A FIT TO ELECTROWEAK DATA

The BBCCV corrections are incorporated in the ZFITTER program [6,7] which is used by the LEP/SLC electroweak working groups to derive parameters such as m_t and m_H from Z data [3]. We make modest modifications to ZFITTER to allow for the effects of V_{tb} described in the previous section.

The Z parameters, from a combined fit to LEP/SLC data, which we use as input are [3]: the Z mass, m_Z; the Z width, Γ_Z; the hadronic pole cross-section, σ_h^0; $R_\ell \equiv \Gamma_{\rm had}/\Gamma_{\ell\ell}$ where $\Gamma_{\rm had}$ is the hadronic partial Z width and $\Gamma_{\ell\ell}$ is the leptonic partial width, assuming lepton universality; and the leptonic pole forward-backward charge asymmetry assuming lepton universality, $A_{\rm FB}^{0,\ell}$. The parameter values, their errors, and their correlation coefficients are shown in Table I.

The parameters pertaining to b and c quarks which we use are [3]: $R_b^0 \equiv \Gamma_{b\bar{b}}/\Gamma_{\rm had}$; $R_c^0 \equiv \Gamma_{c\bar{c}}/\Gamma_{\rm had}$; $A_{\rm FB}^{0,b}$ and $A_{\rm FB}^{0,c}$, the forward-backward charge asymmetries at the Z pole for b and c quarks respectively; and A_f ($f = b, c$) where $A_f \equiv 2g_{fV}g_{fA}/(g_{fV}^2 + g_{fA}^2)$. The parameter values, their errors, and their correlation coefficients are shown in Table II.

We also use the following parameters which are to a good approximation experimentally uncorrelated: $A_\tau \equiv -P_\tau$, the average tau polarisation [3]; A_e from the tau polarisation forward-backward asymmetry [3]; $A_{\rm LR}$, the left-right asymmetry from SLD [3]; the QED coupling constant, $\alpha(m_Z)$ [8]; the strong coupling constant $\alpha_{\rm s}(m_Z)$ [2] where the value obtained from the Z width is not included; the W masses from LEP II [3], CDF [9], DØ [10], and UA2 [11], averaged according to Ref. [12]; the top mass from the Tevatron [4]; and $(1 - m_W^2/m_Z^2)$ from νN scattering measurements by CHARM [13], CDHS [14], and CCFR [15]. The parameter values and their errors are shown in Table III.

For given values of m_Z, m_t, m_H, $\alpha_{\rm s}$, α, and $|V_{tb}|$ our modified version of ZFITTER provides predictions for all of the parameters shown in Tables I, II, and III. These predictions and the corresponding measured quantities, together with their associated errors and correlation coefficients, are used to construct a chisquare probability $\chi^2(m_Z, m_t, m_H, \alpha_{\rm s}, \alpha, V_{tb})$. The minimum of the chisquare is then determined numerically. As a technical cross-check, we use the same input parameters as those of Ward [3], set $|V_{tb}| = 1$, and successfully reproduce the results for m_Z, m_t, m_H, $\alpha_{\rm s}$, and α.

The results of the fit with $|V_{tb}|$ free are shown in Table IV. We determine $|V_{tb}| = 0.77^{+0.18}_{-0.24}$. This value is consistent with the unitarity prediction of $|V_{tb}| \approx 0.9991$ to within approximately one standard deviation. For comparison, Table IV also includes the results with $|V_{tb}|$ fixed to 0.9991. In both fits, the χ^2 probability is consistent with expectations given the number of degrees of freedom.

IV. DISCUSSION

The fitted values of m_Z and $\alpha(m_Z)^{-1}$ are insensitive to $|V_{tb}|$ as expected, as shown by the correlation coefficients from the fit with $|V_{tb}|$ free in Table V. The anti-correlations of $|V_{tb}|$ with m_t and m_H, shown in Fig. 2(a) and 2(b) and in Table V, have only a weak effect on the determinations of m_t and m_H since the Tevatron measurement of m_t and the vacuum polarisation contribution to the Z width constrain m_t and m_H independent of $|V_{tb}|$. Nonetheless, allowing $|V_{tb}|$ to float increases the fitted values of m_t by 1.5 GeV and of m_H by approximately 30 GeV. Allowing $|V_{tb}|$ to float decreases the fitted value of $\alpha_{\rm s}$ by approximately 0.7 standard deviations, due to the fairly strong correlation of these two quantities, as shown in Fig. 2(c) and in Table V.

To assess the future sensitivity of this technique for determining $|V_{tb}|$ we reduce the error by a factor of two on each of the input parameters in turn, without changing the errors on the other parameters. The only parameters which cause $\Delta|V_{tb}|/|V_{tb}|$ to change by a relative amount of more than 10%, from the original value of

$\Delta|V_{tb}|/|V_{tb}| \approx 26.5\%$, are Γ_Z, R_b^0, and $\alpha_s(m_Z)$, which yield uncertainties of $\Delta|V_{tb}|/|V_{tb}| \approx 22.5\%$, 19.9%, and 22.4% respectively. Factors of somewhat less than two may be expected from the final analyses of the LEP I and SLC data samples. We estimate that the error on $|V_{tb}|$ from the final Z samples of the LEP and SLC experiments will be approximately 20%.

Recently CDF presented preliminary results of an analysis of their $t\bar{t}$ event samples. However, they necessarily assumed that there are only three generations and that $|V_{td}|^2+|V_{ts}|^2+|V_{tb}|^2=1$, as required by unitarity, to extract $|V_{tb}|^{3\text{gen}} = 0.99\pm0.15$ [16]. Ultimately, the measurement of the single top quark production rate at hadron colliders should be sensitive to $|V_{tb}|$ without requiring such assumptions of unitarity. The estimated sensitivity at the end of the Tevatron Run II for $\delta|V_{tb}|/|V_{tb}|$ is 12%–19%, depending on the uncertainty of the gluon structure functions [17].

V. SUMMARY

We describe a new technique for the determination of the CKM matrix element $|V_{tb}|$ using loop corrections to electroweak processes, without using any unitarity constraints. From a combined analysis of data from the LEP, SLC, Tevatron and neutrino scattering experiments we determine $|V_{tb}| = 0.77^{+0.18}_{-0.24}$ where the error includes the experimental errors and uncertainties on the top mass, the Higgs mass, and the strong coupling constant. The wider implications of this measurement are discussed elsewhere [18].

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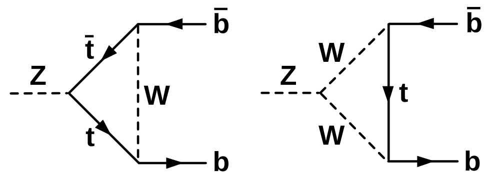

FIG. 1. Vertex correction diagrams, at order one-loop, which contribute to the partial width for $Z \to b\bar{b}$.

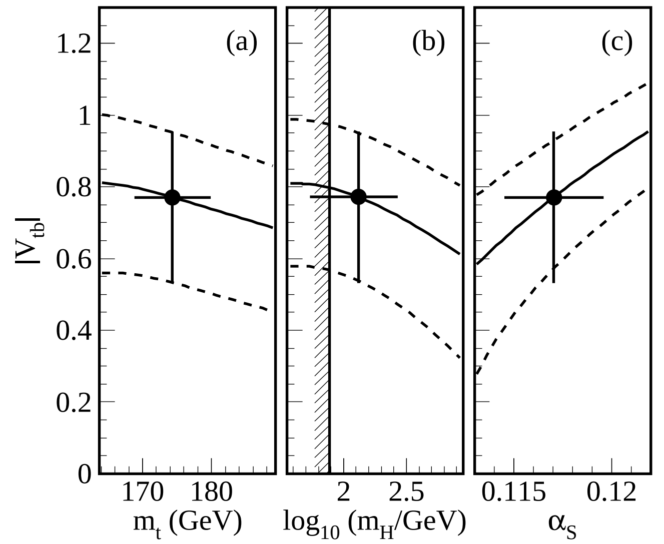

FIG. 2. Variation of $|V_{tb}|$ with (a) m_t, (b) $\log_{10}(m_H/\mathrm{GeV})$, and (c) α_s. The point with error bars denotes the result of the fit allowing for all errors and correlations. The solid line shows the dependence of $|V_{tb}|$ on the ordinate variable; the dashed lines correspond to the 68% confidence level. The hatched line of (b) shows the low m_H region excluded by direct searches at LEP II.

TABLE I. Measured values and the correlation matrix for m_Z, Γ_Z, σ_h^0, R_ℓ, and $A_{\rm FB}^{0,\ell}$ from a combined fit to LEP and SLC data.

	Measured value	Correlation coefficient				
		m_Z	Γ_Z	σ_h^0	R_ℓ	$A_{\rm FB}^{0,\ell}$
m_Z (GeV)	91.1867 ± 0.0020	1.00	0.05	−0.01	−0.02	0.06
Γ_Z (GeV)	2.4948 ± 0.0025	0.05	1.00	−0.16	0.00	0.00
σ_h^0 (nb)	41.486 ± 0.053	−0.01	−0.16	1.00	0.14	0.00
R_ℓ	20.775 ± 0.027	−0.02	0.00	0.14	1.00	0.01
$A_{\rm FB}^{0,\ell}$	0.0171 ± 0.0010	0.06	0.00	0.00	0.01	1.00

TABLE II. Measured values and the correlation matrix for R_b^0, R_c^0, $A_{\rm FB}^{0,b}$, $A_{\rm FB}^{0,c}$, A_b, and A_c from a combined fit to LEP and SLC data.

	Measured value	Correlation coefficient					
		R_b^0	R_c^0	$A_{\rm FB}^{0,b}$	$A_{\rm FB}^{0,c}$	A_b	A_c
R_b^0	0.2170 ± 0.0009	1.00	−0.20	−0.03	0.01	−0.03	0.02
R_c^0	0.1734 ± 0.0048	−0.20	1.00	0.03	−0.07	0.04	−0.04
$A_{\rm FB}^{0,b}$	0.0984 ± 0.0024	−0.03	0.03	1.00	0.13	0.03	0.02
$A_{\rm FB}^{0,c}$	0.0741 ± 0.0048	0.01	−0.07	0.13	1.00	0.00	0.07
A_b	0.900 ± 0.050	−0.03	0.04	0.03	0.00	1.00	0.08
A_c	0.650 ± 0.058	0.02	−0.04	0.02	0.07	0.08	1.00

TABLE III. Measured values of uncorrelated parameters used in our fits.

	Measured value
A_τ	0.1410 ± 0.0064
A_e	0.1399 ± 0.0073
$A_{\rm LR}$	0.1547 ± 0.0032
$\alpha^{-1}(m_Z)$	128.896 ± 0.090
$\alpha_s(m_Z)$	0.118 ± 0.003
m_W (GeV)	80.400 ± 0.075
m_t (GeV)	175.6 ± 5.5
$1 - m_W^2/m_Z^2$	0.2254 ± 0.0037

TABLE IV. Results of the fit for m_Z, m_t, $\log_{10}(m_H/\mathrm{GeV})$, $\alpha_s(m_Z)$, $\alpha(m_Z)^{-1}$, and $|V_{tb}|$ (second column). For comparison, the third column shows the results of the fit with the constraint of $|V_{tb}| \equiv 1$. P denotes the probability of obtaining a reduced chisquare greater than that from the fit.

	$\lvert V_{tb}\rvert$ free	$\lvert V_{tb}\rvert$ fixed
m_Z (GeV)	91.1866 ± 0.0020	91.1866 ± 0.0020
m_t (GeV)	174.2 ± 5.4	172.7 ± 5.2
$\log_{10}(m_H/\mathrm{GeV})$	$2.15^{+0.30}_{-0.39}$	$2.04^{+0.30}_{-0.37}$
$\alpha_s(m_Z)$	0.1171 ± 0.0025	0.1188 ± 0.0021
$\alpha^{-1}(m_Z)$	128.913 ± 0.092	128.905 ± 0.091
$\lvert V_{tb}\rvert$	$0.77^{+0.18}_{-0.24}$	0.9991 (fixed)
$\widetilde{\chi}_0^2 \equiv \chi^2/\mathrm{d.o.f}$	$15.1/(19-6)$	$16.7/(19-5)$
$P(\widetilde{\chi}^2 > \widetilde{\chi}_0^2)$ (%)	30	27

TABLE V. Correlation coefficients from the fit for m_Z, m_t, $\log_{10}(m_H/\mathrm{GeV})$, $\alpha_s(m_Z)$, $\alpha(m_Z)^{-1}$, and $|V_{tb}|$.

	m_Z	m_t	$\log_{10}(m_H)$	α_s	α^{-1}	$\lvert V_{tb}\rvert$
m_Z	1.00	0.01	0.04	−0.01	0.01	0.01
m_t	0.01	1.00	0.65	−0.03	0.16	−0.15
$\log_{10}(m_H)$	0.04	0.65	1.00	0.04	0.65	−0.23
α_s	−0.01	−0.03	0.04	1.00	0.03	0.52
α^{-1}	0.01	0.16	0.65	0.03	1.00	−0.08
$\lvert V_{tb}\rvert$	0.01	−0.15	−0.23	0.52	−0.08	1.00